Readers' Forum

Brief discussions of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A Discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Second-Order Accurate Boundary Conditions for Compressible Flows"

G. Moretti*
G.M.A.F. Inc., Freeport, New York

THE article in question[1] is about 15 years outdated, despite its reference to a few recent papers. The statement that "the method commonly used to apply boundary conditions on a solid surface is to consider additional mesh points at the interior of the body" was true in the 1960's but not any more. The figures published in support of the author's technique are wrong. The many excellent codes (see, e.g., Ref. 2) that we have at our disposal today show that the flow about a circle at $M_\infty = 0.4$ is barely critical at $\theta = 90$ deg and is perfectly symmetrical. None of the curves of Fig. 3 (at $M_\infty = 0.35$) is even symmetrical, and Figs. 4 and 5 show a supersonic region at $M_\infty = 0.4$ that does not exist. Again, whether such poor results depend on a poor code or merely on a programming error, they show that the author may not be aware of the calculations performed in the last 15 years with second-order accuracy.

References

[1]Sparis, P. D., "Second-Order Accurate Boundary Conditions for Compressible Flows," *AIAA Journal*, Vol. 22, Sept. 1984, pp. 1222-1228.

[2]Jameson, A., "Transonic Potential Fow Calculations on Arbitrary Meshes by the Multiple Grid Method," *Proceedings, AIAA Computational Fluid Dynamics Conference*, July 1979, pp. 122-146.

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*Consultant. Fellow AIAA.

Reply by Author to G. Moretti

P. D Sparis*
Polytechnic School of Thrace, Xanthi, Greece

THE purpose of the present paper[1] was to amplify the negative effects of lower order accurate boundary conditions, as the so-called "reflection principle," on the solution accuracy. Dr. Moretti will be surprised to realize the number of people still using the "reflection principle" today, in spite of his early warnings[2] 15 years ago. For these people, the paper offers an improved version of the "reflection principle" that can be very easily implemented in the existing codes using dummy points, and can offer second-order accuracy incorporating the effect of the local radius of curvature, that is, a first-order effect.

As far as the lack of symmetry of the $M_\infty = 0.35$ solution, it should be clear to a CFD analyst that, due to the symmetry, it cannot be caused by the boundary conditions, but most probably by the action of excessively strong artificial viscosity terms. However, since the same terms were used in all cases, the comparison is rather fair. The important result illustrated in Fig. 3 is that the lower order accurate boundary condition methods yield reduced maximum velocities at $\theta = 90$ deg.

Finally, I do not quite understand Dr. Moretti's argument with respect to the $M_\infty = 0.4$ solution. The proposed method predicts that for $M_\infty = 0.4$ there is a small supersonic region that extends for a very small number of mesh points in the r and θ directions, while other codes predict that for the same Mach number the flow is barely critical. In the author's opinion, these two results are in good agreement in view of the coarseness of the mesh. The supersonic region in my computation extends in the radial direction only for a distance of two mesh points. Although I am not familiar with the boundary condition treatment of Dr. Moretti's codes, the appearance of lower flow velocities might be an indication of a lower boundary condition accuracy in the excellent codes developed in the last 15 years.

References

[1]Sparis, P. D., "Second-Order Accurate Boundary Conditions for Compressible Flows," *AIAA Journal*, Vol. 22, Sept. 1984, pp. 1222-1228.

[2]Moretti, G., "Importance of Boundary Conditions in the Numerical Treatment of Hyperbolic Equations," *The Physics of Fluids*, Supp. II, Dec. 1969, pp. 11-20.

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*Professor of Mechanical Engineering. Member AIAA.

Comment on "A Class of Bidiagonal Schemes for the Euler Equation"

Gérard Degrez*
Université Libre de Bruxelles
Brussels, Belgium

THE class of bidiagonal schemes introduced in Ref. 1 seems to be a very promising class of methods for solving the Euler equations and, as mentioned by the authors, includes both MacCormack's explicit and implicit schemes[2,3] as particular choices of parameters.

The explicit MacCormack scheme corresponds to $\theta = 0$, $\xi = ½$. However, the identification of parameters for Mac-

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*Assistant, Institute of Aeronautics.

Cormack's implicit scheme made in the original paper, although algebraically correct, does not seem to be the most suitable. Due to its resemblance to the explicit scheme, the following parameters are proposed: $\xi = \frac{1}{2}$, θ is related to the parameter λ introduced in the MacCormack scheme [Ref. 3, Eq. (5)] by $\theta = \lambda / |a|$. With MacCormack's choice,

$$\lambda = \max\left\{ |a| - \frac{1}{2\sigma},\ 0 \right\}$$

This yields

$$\theta = \max\left\{ 1 - \frac{1}{2\sigma |a|},\ 0 \right\}$$

This choice of parameters appears more convenient in the sense that when the implicit option is not used in MacCormack's scheme ($\lambda = 0$), the parameter θ is zero.

Finally, since according to the stability analysis in Ref. 1, the optimum choice for the dissipation of numerical modes is, in this case,

$$\theta = \frac{\sqrt{2}}{2} - \frac{1}{2\sigma |a|}$$

the optimum choice for a scheme taking advantage of the explicit option whenever possible from stability considerations would be

$$\theta = \max\left\{ \frac{\sqrt{2}}{2} - \frac{1}{2\sigma |a|},\ 0 \right\}$$

References

[1]Casier, F., Deconinck, H., and Hirsch, Ch., "A Class of Bidiagonal Schemes for Solving the Euler Equations," *AIAA Journal*, Vol. 22, Nov. 1984, pp. 1556-1563.

[2]MacCormack, R. W., "The Effect of Viscosity in Hypervelocity Impact Cratering," AIAA Paper 69-354, April 1969.

[3]MacCormack, R. W., "A Numerical Method for Solving the Equations of Compressible Viscous Flow," *AIAA Journal*, Vol. 20, Sept. 1982, pp. 1275-1281.

Reply by Authors to G. Degrez

Herman Deconinck* and Charles Hirsch†
Vrije Universiteit Brussel, Brussels, Belgium

THE authors[1] generally agree with this Comment. Indeed, the central bidiagonal scheme (CBS) contains two parameters (θ and ξ), whereas the McCormack implicit scheme contains only one parameter (λ). This allows the free choice of one of the CBS parameters (e.g., ξ) for the identification of both schemes.

The choice of $\xi = \frac{1}{2}$ made by Degrez is indeed preferable if one wishes to compare or to switch to the explicit McCormack scheme obtained for $\xi = \frac{1}{2}$, $\theta = 0$. In particular, this choice allows the traditional interpretation of the space derivatives in the McCormack implicit scheme as one-sided downwind in the U-sweep predictor step [discretized at $i + \frac{1}{2} - \xi = i$ using cell $(i, i+1)$] and one-sided upwind in the L-sweep corrector step [discretized at $i + \frac{1}{2} + \xi = i+1$ using cell $(i, i+1)$]. As a result, each step is first-order accurate in space. Note that the McCormack scheme as described in Ref. 2 of the Comment is a U-L scheme with sweeps from right to left in the predictor and left to right in the corrector step, explaining the reversed sign of ξ compared to the L-U scheme described in our paper.[1]

In the context of the paper, however, the choice $\xi = 0$ leading to $\theta = 1$ has some attractive aspects: The type of calculations made by the authors uses a constant local CFL number in each meshpoint, equal to the value specified in the data file. Hence, the explicit stability condition is never satisfied in any point of the mesh and application of the McCormack implicit scheme would never switch to the explicit scheme in this case. Thus, the choice of $\theta = 1$ is independent of the CFL number, as opposed to the choice made in the Comment.

Further, the choice $\xi = 0$, $\theta = 1$ shows the close resemblance with the optimal fully implicit scheme used in the numerical tests in the paper and determined by the choice $\xi = 0$, $\theta = \sqrt{2}/2 = \pm 0.7$.

Finally, the choice $\xi = 0$, $\theta = 1$ shows that the McCormack implicit scheme can be interpreted as resulting from a central second-order discretization in the point $i + \frac{1}{2}$ in both predictor and corrector steps (if no use is made of the explicit option). This central "box" interpretation of the McCormack implicit scheme strongly differs from the usual interpretation and can have important consequences, e.g., in the presence of a source term as in the quasi-one-dimensional Euler equations. The source term for the scheme $\xi = \frac{1}{2}$ would be discretized at i in the predictor step and at $i+1$ in the corrector step, precisely as in the traditional explicit McCormack scheme. For the scheme $\xi = 0$, however, the source term would be discretized at $i + \frac{1}{2}$ in both the predictor and corrector step,[1] which is obtained by taking the average over the values at i and $i+1$. On the other hand, with $\xi = 0$, each step is second-order accurate in space at $i + \frac{1}{2}$.

Again, the identification proposed by Degrez is more in line with the traditional McCormack approach.

References

[1]Casier, F., Deconinck, H., and Hirsch, Ch., "A Class of Bidiagonal Schemes for Solving the Euler Equations," *AIAA Journal*, Vol. 22, Nov. 1984, pp. 1556-1563.

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*Research Assistant, Department of Fluid Mechanics.

†Professor, Department of Fluid Mechanics.

Comment on "Application of the Generalized Inverse in Structural System Identification"

John A. Brandon*
University of Wales Institute of Science and Technology Cardiff, Wales, U.K.

THE use of the Moore-Penrose generalized inverse underlies much of the current work in structural dynamics. Whereas Chen and Fuh[1] and Berman[2,3] use this inverse explicitly (and correctly) in model adjustment using identified modes, it is often also used implicitly in the identification algorithms that provide the data for model adjustment. In this application it is not always clear that the analyst is aware of the limitations of the method. The essential difference is that Chen and Fuh[1] are able to assume in their analysis that "the measured modal matrix Φ $(n \times m)$ is rectangular with full column rank m." In the identification stage, however, the rank of the modal matrix corresponds to the number of identifiable modes in the test data. Under many test conditions the decision as to the number of identifiable modes is not clear-cut and depends on the subjective judgment of the analyst.

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*Lecturer, Department of Mechanical Engineering and Engineering Production.